Doppler Distortions Eliminated

Do you believe that a perfectly linear loudspeaker doesn't generate distortions?
That is wrong. Here is why.
Even a perfectly linear loudspeaker produces two types of distortions because of the movement of its membrane. First Doppler distortions, a.k.a. Phase Modulation, and second distortions stemming from the Nonlinear Amplitude Modulation.
Doppler distortions generate non-harmonic spectral lines, which are independent from the listening distance and need to be eliminated for a good listening experience.
The Nonlinear Amplitude Modulation generates non-harmonic spectral lines too, but decreasing with listening distance and that is a problem only at short listening distances.
This article characterizes the Doppler distortions and shows how to eliminate them.

A perfect sound reproduction requires a system able to translate the sound pressure of a source into a usually storable signal (e.g. an electrical audio signal) and translates that signal back into sound pressure without modification.
In order to do that system theory tells that one needs **first** a **linear** and **second** a **time invariant** system.

One can fulfill the first requirement on linearity using a loudspeaker e.g. with Acceleration Feed Back (AFB) as described in [1]. By this technique the linearity of a loudspeaker can be improved such, that harmonic and intermodulation distortions are reduced to below -60 dB (0.1%) relative to the fundamental frequency.
Figure 1 shows the Sound Pressure Level (SPL) frequency response of an acceleration-controlled loudspeaker together with the harmonic distortions k2 and k3.

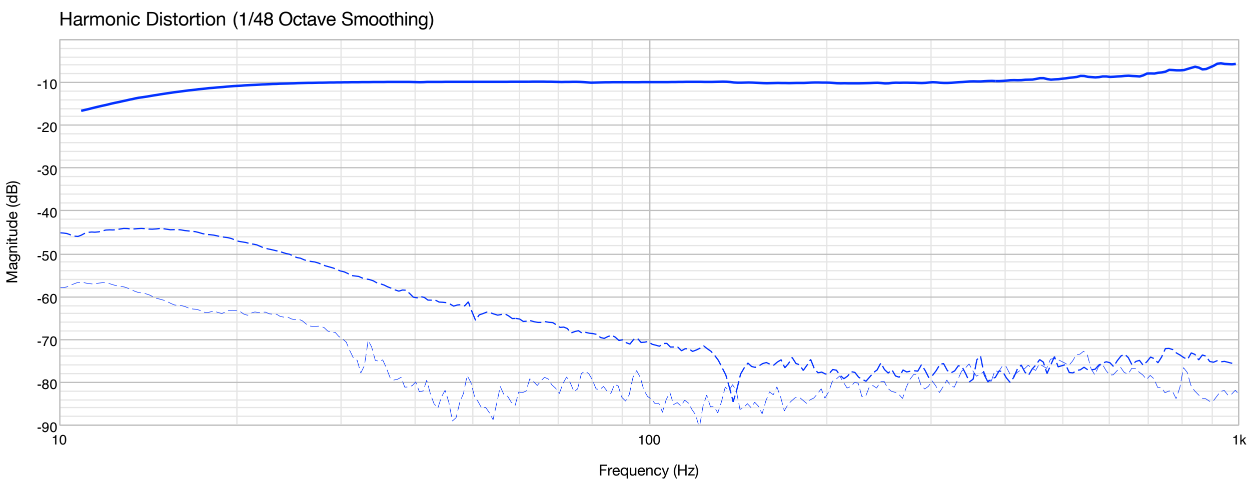


Figure 1: SPL Frequency Response of an Acceleration Controlled Loudspeaker SEAS L15 RLY/P with an AudioChiemgau AC-PAR75 Amplifier. Solid blue SPL amplitude, upper dashed line harmonic distortion k2 and lower dashed line Harmonic distortion k3.
Measurement parameters: AudioChiemgau ModeCompensator, Sweep Time 10 Seconds, Time Window 10 Seconds, Frequency resolution 0.1 Hz. The stimulus signal is High Pass filtered at 15 Hz (-3 dB) in order to limit the membrane amplitudes at extremely low frequencies (< 15 Hz).

Obviously, with acceleration control technology, a sufficient linear system can be achieved with moderate effort.
Above 100 Hz the harmonic distortions are below -60 dB (0.1 %). However, at lower frequencies especially the harmonic k2 is rising towards lower frequencies.

The reason lies inter alia in the violation of the second above mentioned **requirement on time invariance**.
During recording of the sound, the membrane of a condenser microphone moves with an amplitude of about 1 μm at low frequencies (20 Hz) and high sound pressure levels (100 dB). The same is valid for the tympanic membrane of the human ear.
However, during play back a loudspeaker membrane may move up to 1 cm amplitude in order to reproduce the recorded sound at the above mentioned frequency and sound pressure level.

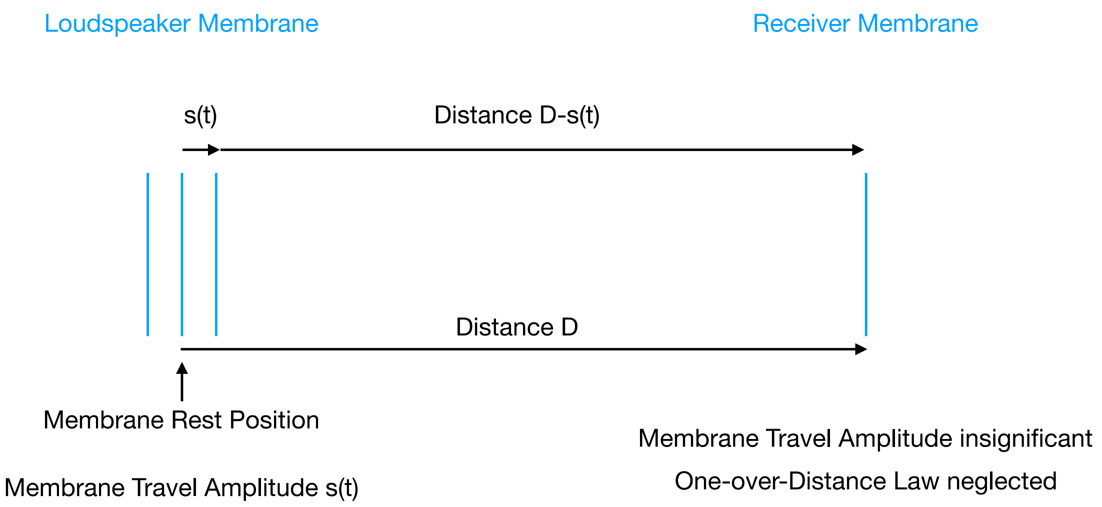
To put it in simple terms: We record at an almost fixed location in the sound space, but we reproduce at a variable location in the sound space. This 1 to 10000 relation of the membrane amplitudes between recording and reproduction leads to well audible distortions, even for a perfect linear loudspeaker.

Phase Modulation of the Moving Membrane

In the following example the distance change (excursion, or travel) of the membrane is assumed to be ± 7 mm from the rest position. The travel amplitudes of the membrane and the attached air molecules during playback generate an undesired Phase Modulation [2] also known as Doppler Distortion, or Doppler Effect of the Sound Pressure at the receiver (microphone or ear).

Nota bene: The sound pressure is strictly proportional to the membrane acceleration in the frequency range of interest.

Figure 2 illustrates how the membrane excursion is translated into a phase modulation.



$s(t) := S*\cos(2*\pi*f*t)$

$al := -S*(2*\pi*f)^\wedge 2*\cos(2*\pi*f*t)$

$ar(t) := -S*(2*\pi*f)^\wedge 2*\cos(2*\pi*f*(t-(D-s(t))/c)$

Figure 2: Varying membrane positions of the transmitter (loudspeaker membrane) lead to varying travel times to the steady receiver. The time function s(t) describes the membrane travel, al(t) describes the membrane acceleration at the transmitter and ar(t) is the receiver signal proportional to the received sound pressure.

Comparing the waveform at the receiver with that of the transmitter shows the delay caused by the travel time over the distance of e.g. D=1 Meter (left image). The small delay variations caused by the membrane movement s(t) become only visible when the receiver waveform is compared to the ideal receiver waveform without s(t), (right image green waveform).

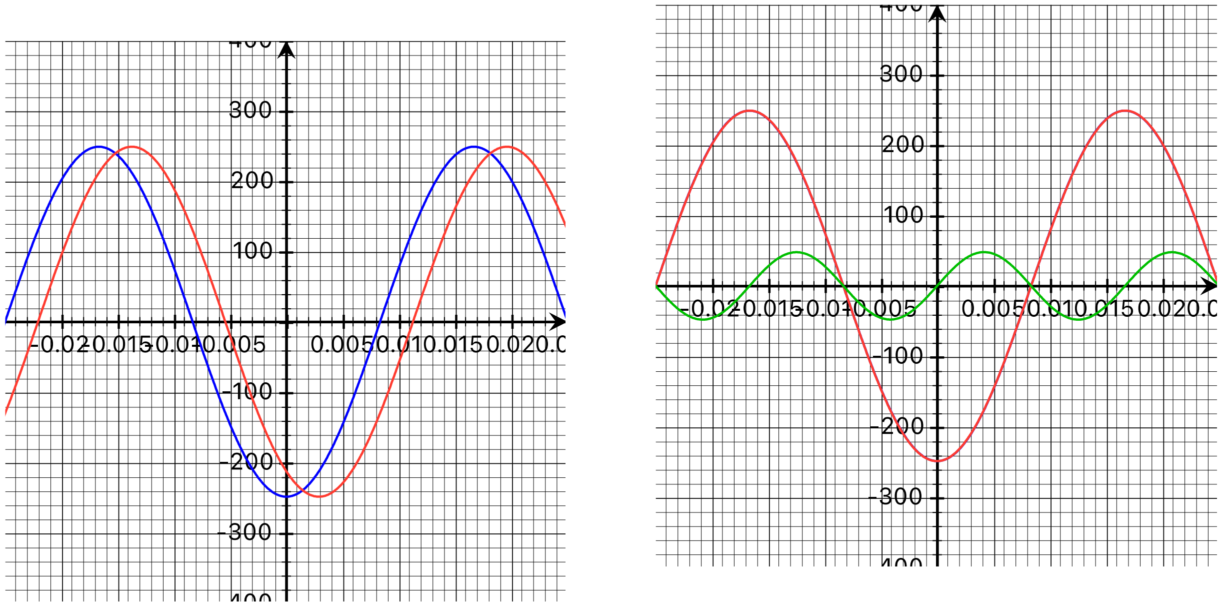


Figure 3: Left image: Delayed waveform (red) for one Meter travel distance compared to the transmitter waveform (blue). Right image: Difference (multiplied by 100) between the receiver waveform with and without the variable delay caused by the membrane movement. Axes: Acceleration amplitudes [m*s^-2] versus time [s].

For the further discussion the constant delay caused by the distance D is of no interest. The delay variations by the movement of the transmitter membrane, however, generate a phase modulation, which in turn generates additional spectral lines as we will see later. The delay s(t)/c shown in Figure 1 equals multiplied with 2*π*f to the frequency depending phase contribution p(t) of the receiver waveform ar(t). The equation for the receiver waveform can therefore be rewritten

$$ar(t)=-S*(2*\pi*f)^2*\cos(2*\pi*f*t + p(t))$$

with the phase term

$$p(t):=s(t)*f/c*2*\pi$$

Comment: Without knowledge of the modulating frequency a Phase Modulation cannot be differentiated from a Frequency Modulation. Therefore the Doppler effect is sometimes said to be a frequency modulation. However the formulas above show, that the moving loudspeaker membrane generates an undesired Phase Modulation.

Thought Experiment:

Imagine a high note being played back by a perfectly linear loudspeaker. Simultaneously a low note with considerable membrane amplitude is being played back. The location at which the high note is generated travels with the membrane back and forth. Because of the distance changes the sound needs different times to arrive at the receiver. Such different membrane positions s(t) translate into different arrival times at the receiver and depending on the frequency f to different phases p(t) at the receiver according to the above formula for p(t).

This principle is a general one and valid for any frequency, even, when only a single frequency is reproduced.

According to the above formula, the phase of the sound pressure is proportional to the membrane travel s(t) and to the frequency f. The speed c of the sound is 343 m/s under normal conditions.

At constant sound pressure level, the membrane excursion amplitude is quadratic invers to the frequency following the Inverse-Square Law [3]. s(t) in the nominator of the formula above increases quadratically towards lower frequencies. However, also the frequency f is standing in the nominator falling linearly towards lower frequencies. That compensates partially the quadratic rise of the membrane amplitude towards lower frequencies. Therefore, the phase deviation is growing linearly towards lower frequencies at a given constant SPL, i.e. with a slope rising at 20 dB per frequency decade towards lower frequencies. That behavior can be seen in Figure 1 for the second harmonic.

Phase Modulation in the Time and Frequency Domain

The effect of modulating the phase of a signal is well known. For the listener it has the consequence that additional and especially non-harmonic spectral lines are generated and added to the original sound / to the original spectrum.

In a time domain formulation f is the frequency, S the amplitude and st(t) the membrane and attached air molecule excursion at the transmitter.

The following example uses f = 30 Hz at S = 7 mm membrane travel amplitude. al(t) is the acceleration of the loudspeaker membrane and the attached air molecules, the acceleration being strictly proportional to the sound pressure.

The following equations use the the mks (Meter, Kilogram, Second) System.

f=30
S= 7e-3
sl(*t*)=*S**cos(2*π**f***t*)
al=-*S**(2*π**f*)^2*cos(2*π**f***t*)

Due to the membrane movement the distance to the locally constant receiver varies with time resulting in the phase pr(t).

c=343
pr(*t*)=*s*(*t*)**f*/*c**2*π

Such the waveform at the receiver becomes

ar(*t*)=-S*(2*π**f*)^2*cos(2*π**f***t*+*pr*(*t*))

It is obvious from Figure 3 (right image) that the waveform generated by the phase modulation contains additional frequencies, also with twice the frequency of the original waveform shifted 90 degrees in phase.

The Fourier Transformation of the waveform at the receiver in Figure 4 shows that there are further harmonic frequencies generated, however, with significant lower amplitudes.

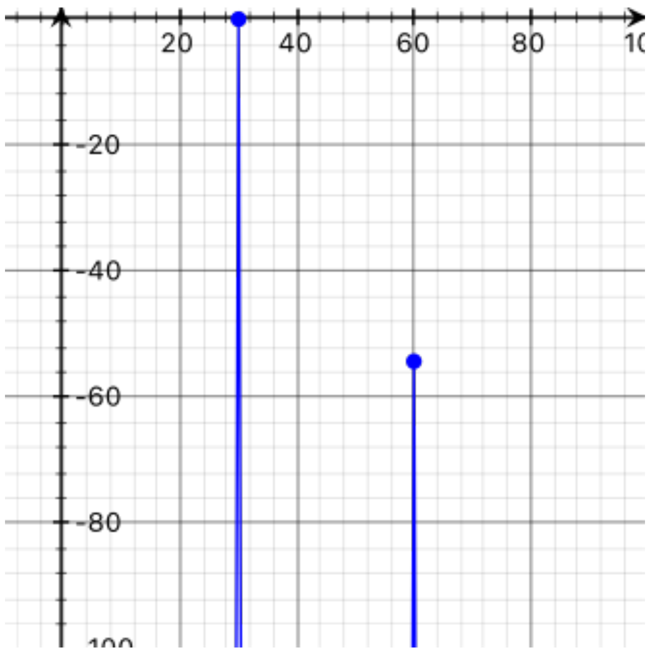


Figure 4: Frequency domain display of the spectral lines generated by the Phase Modulation.

X axis frequency in Hz, Y axis normalized SPL [dB]. The third harmonic is e.g. below -100 dB.

Figure 4 shows a dominant second harmonic to the fundamental frequency at about -54 dB.

One could now think: Not nice, but not significant disturbing.

However, as soon as more than one frequency is reproduced - and we are rarely listening to single sine tones - the phase modulation creates the well-known Bessel spectral lines. The frequency components of the Bessel lines are non-harmonic and severely disturb a good listening experience.

For the calculation in the time domain a second excursion signal with a frequency of 900 Hz is added to the first one with 30 Hz.

Its amplitude A2 is determined to deliver the same acceleration (i.e. sound pressure) as the first frequency.

$$s(t) = A1 * \cos(2 * \pi * f1 * t + p1(t)) + A2 * \cos(2 * \pi * f2 * t + p2(t))$$

The frequency domain display in Figure 5 shows the Bessel lines centered at the second frequency of 900 Hz with the same SPL as the 30 Hz frequency component.

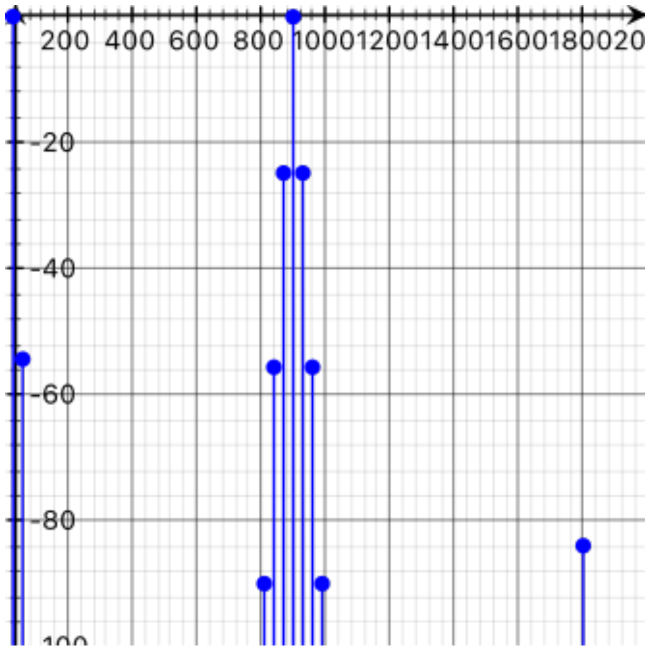


Figure 5: Additional spectral lines, a.k.a. Bessel Lines, generated by two frequencies, the first as before 30 Hz with 7 mm membrane travel amplitude and the second one with 900 Hz and the same SPL as the first one. Further weaker Bessel spectra appear at twice the second frequency a.s.o.

The largest two Bessel Lines at 900 Hz ± 30 Hz are at -24,7 dB (5.8 %).

Further spectral lines appear at 900 Hz ± n times 30 Hz (n being an integer number).

The Bessel Functions of the first kind [4] describe the amplitudes of the generated spectral lines.

The above spectrum would be generated by a two-way loudspeaker system **with a perfect linear chassis** and a cross over frequency of 1000 Hz when reproducing the two tones. The Bessel lines lay non-harmonic and show considerable amplitudes.

The only way to get rid of these disturbing additional frequencies is using an acoustically steady membrane as explained later.

As an approximation to that requirement, a very large membrane area can be used, which reduces the membrane excursion accordingly. That, however, conflicts with a sufficient large dispersion angle at higher frequencies. The usual approximation to this problem in high end loudspeakers: One uses several large area bass chassis with very low crossover frequency. Such the membrane amplitude and consequently the phase modulation is reduced and the frequency band creating harmonic distortions and the Bessel spectra is limited too. As said – an approximation, however, not a solution to the problem.

Consequences for near field measurements of loudspeakers

The second harmonic in the above example is -54. dB below the fundamental frequency of 30 Hz. Other than the Nonlinear Amplitude Modulation (described in another article) the Phase Modulation is independent from the distance to the measurement microphone / listener. In the above case the measurement dynamics would be limited at 30 Hz to -54 dB for the second harmonic. The dependency of the second harmonic from the membrane amplitude is visualized in Figure 6.

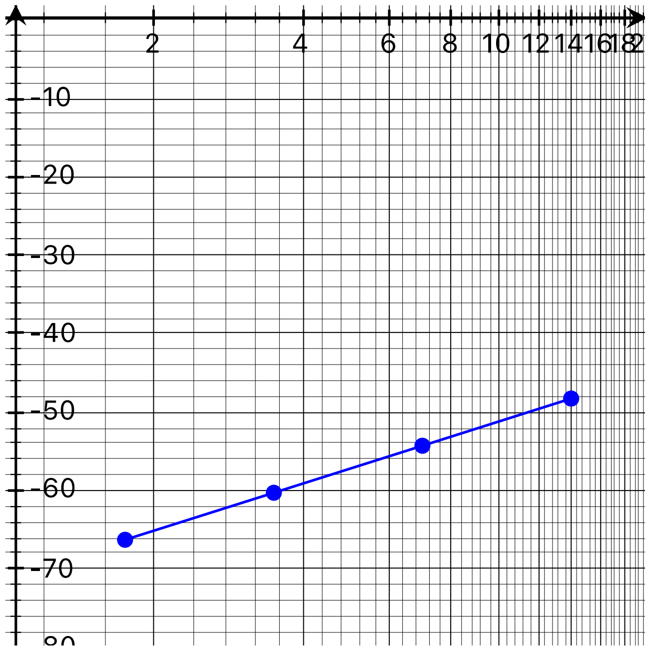


Figure 6: Doubling of the second harmonic (+6 dB) results for each doubling of the membrane amplitude. Axes: dB versus total membrane amplitude in mm.

Consequences for the listening experience

Figure 7 shows the amplitudes of the dominating Bessel lines as function of the second frequency (the first frequency stays at 30 Hz).

At the reproduction of 30 Hz with 7 mm membrane excursion amplitude and a second frequency of 900 Hz with the same sound pressure level, non-harmonic distortions at -24 dB (6.3 %) are visible. Obviously, the level of Bessel distortions can be reduced limiting the frequency band. However only 6 dB can be gained for halving the upper frequency limit.

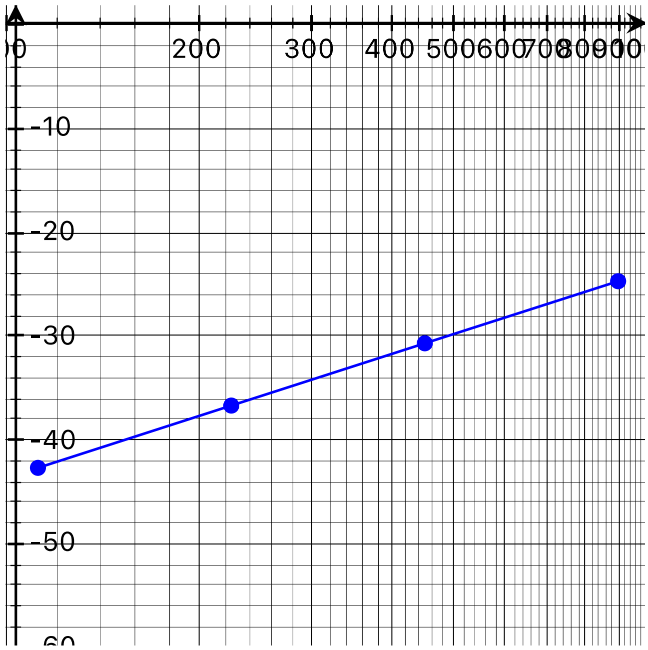


Figure 7: SPL amplitudes of the dominant Doppler Lines with respect to the SPL of the fundamental frequency as function of the second frequency. The chart illustrates the frequency dependency of the Phase Modulation / the Doppler effect.

X axis: Second frequency in Hz,
Fundamental frequency is 30 Hz @ 7 mm amplitude.

Y axis dB SPL of dominant Doppler Lines (Bessel Lines) relative to the SPL of the fundamental frequency.

Characteristics of the Doppler Spectral Lines:

- **As follows from the first formula, the relative Doppler line amplitudes are proportional to the frequency, as the phase variation is proportional to it.**
- **The Doppler line amplitudes are proportional to the total membrane excursion amplitude, which is usually mainly determined by the low frequency components.**
- **The Doppler amplitudes are independent from the measurement microphone / hearing distance.**

The Phase Modulation shows the well known Bessel spectrum in acoustic measurements.

In Figure 8 the low frequency spectral line is at 30 Hz and the high frequency spectral line is at 900 Hz surrounded by the non-harmonic Bessel spectral lines. In order to isolate the Bessel spectrum from e.g. the Non-Linear Amplitude Modulation, a larger microphone distance to the chassis needs to be chosen. Otherwise the spectral lines of the Non-Linear Amplitude Modulation, which have the same frequency components, but are 90 degrees shifted in phase blur the measurement. This can be achieved using the AudioChiemgau ModeCompensator.

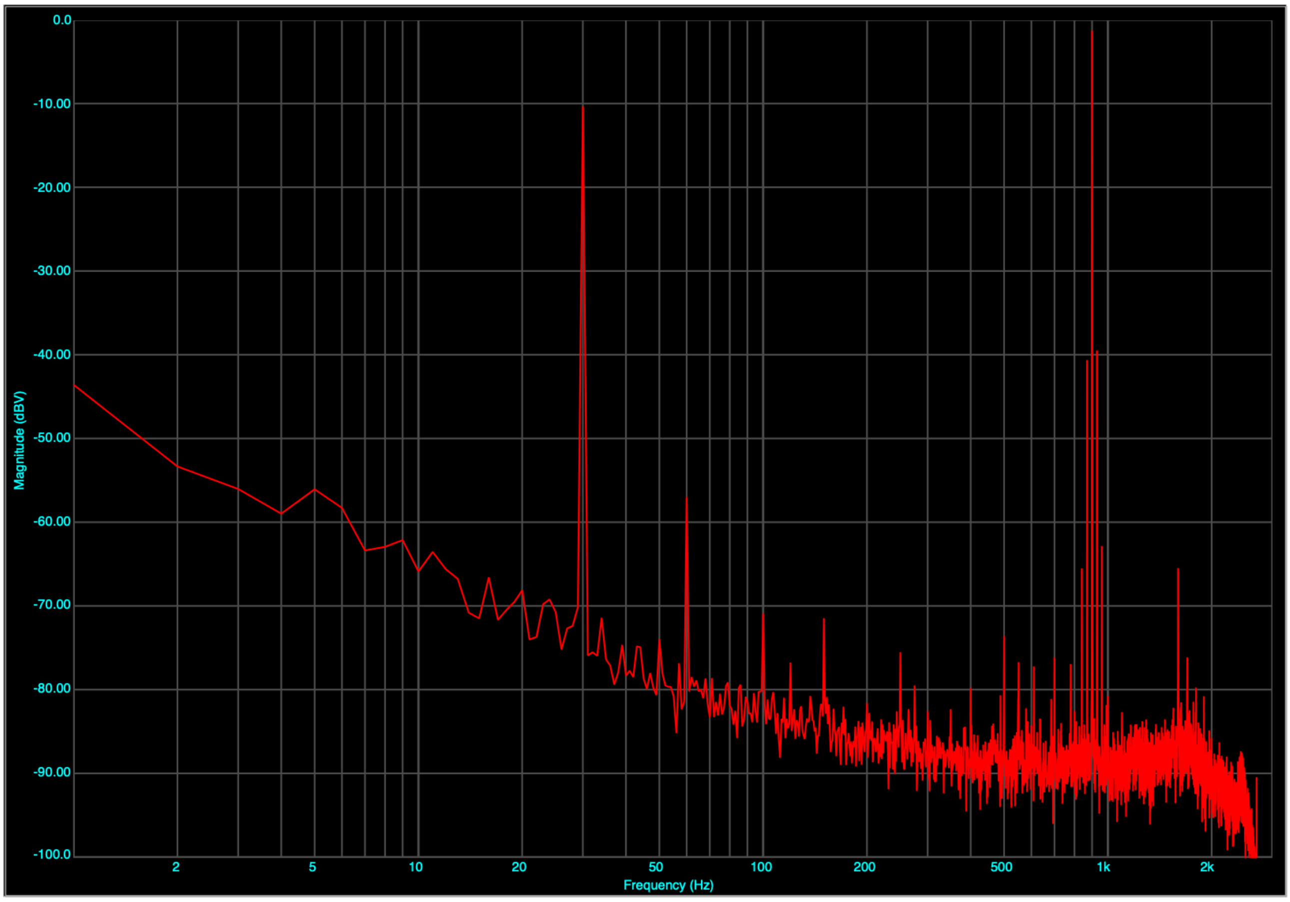


Figure 8: Measured SPL spectrum for 30 Hz and 900 Hz. The second harmonic in this example is clearly visible about 47 dB below the fundamental frequency of 30 Hz. The dominant Bessel lines on both sides of the 900 Hz spectral line are about -37 dB below the central spectral line as result of the phase modulation / the Doppler distortion.

Elimination of the Phase Modulation / the Doppler Effect

As the generation of the PM / the Doppler effect is a deterministic physical effect, it can be compensated independently of the listening distance. According to the formula shown a compensation of the phase deviation can be achieved in case the actual position of the loudspeaker membrane is known at any point in time. The compensation is realized with a phase modulator [5] adding an equal, but opposite sign phase using the membrane position as control signal. Such it is possible to create an acoustically steady membrane, i.e. the acoustical center stands still, even when the membrane moves. This can be achieved with very little effort using acceleration-controlled loudspeakers, as they show a constant SPL amplitude and phase frequency response, which allows the continuous determination of the membrane position with high accuracy.

Example: When the membrane moves towards the listener, such shortening the acoustic path length, the audio signal is electronically retarded by the phase modulator / delay generator such, that the resulting path length remains constant. So, from an acoustical point of view the membrane doesn't move. Without movement of the acoustical center there is no Doppler effect / no Phase Modulation.

Figure 9 shows the same spectrum as in Figure 8 with active Doppler Compensation. The undesired Phase Modulation is reduced to a level of -65 dB with respect to the fundamental frequency which corresponds to a reduction of more than 25dB. Other intermodulation products and the spectral lines of the Non-Linear Amplitude Modulation dominate now the spectrum around the 900 Hz line.

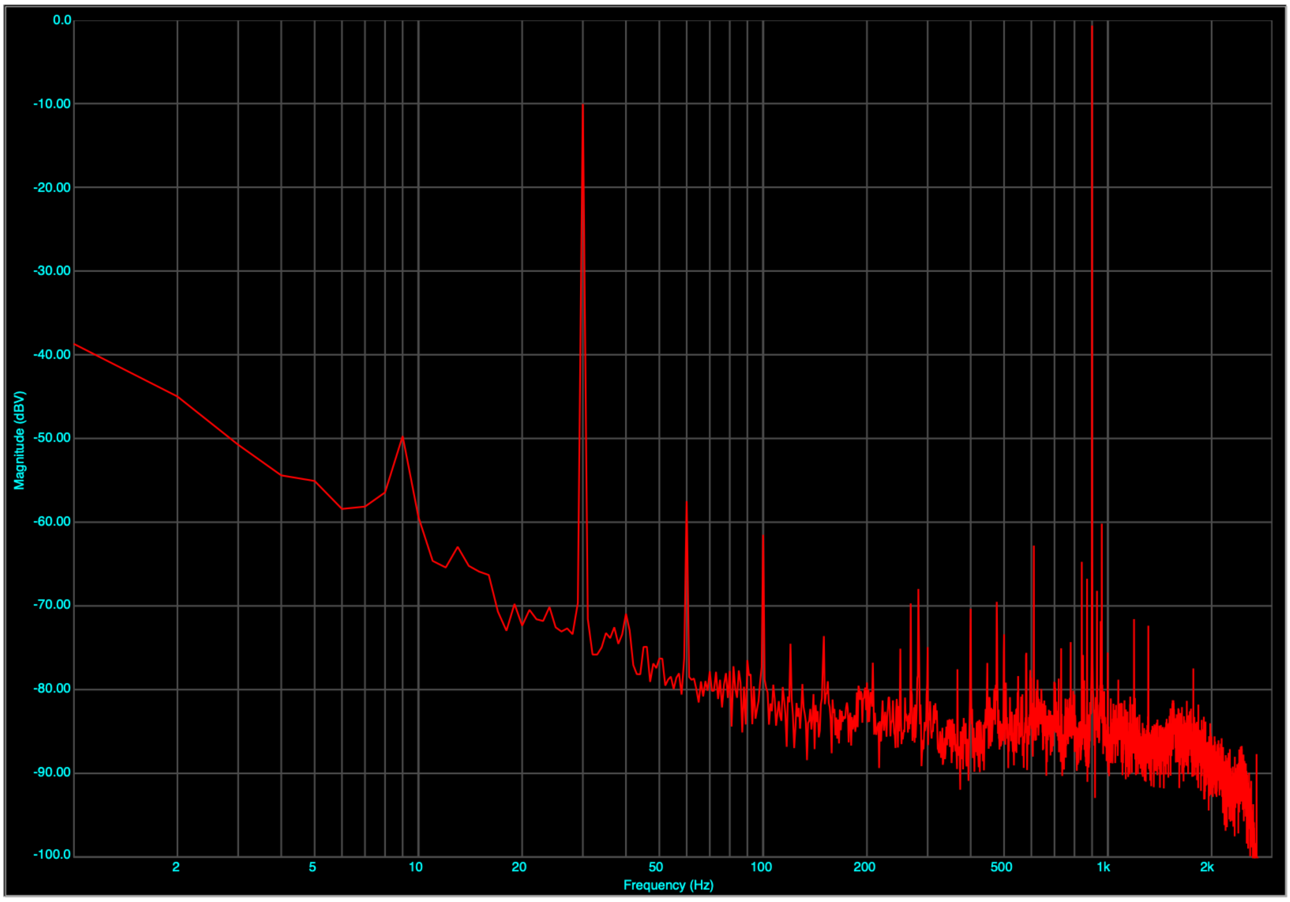


Figure 9: Same spectrum as in Figure 8, however with active Doppler Compensation. The two dominant Bessel lines are down at about -65 dB.

Summary: The Doppler spectral lines, which are non-harmonic to the fundamental frequencies and are therefore very disturbing for a good listening experience can practically be eliminated using the technique of an acoustically steady loudspeaker membrane. A linear analogue Phase Modulator or a digital variable delay, both controlled by the membrane position, are used for this purpose. Both alternatives are realized in the Audio Processors of AudioChiemgau [1].

References:

1. Web page www.AudioChiemgau.de

2. Angle Modulation Part 2: https://web.stanford.edu › class › ee179 › lectures › notes09.pdf

3. Damping of sound level (decibel dB) vs. distance: http://www.sengpielaudio.com/calculator-distance.htm

4. https://de.wikipedia.org/wiki/Bessel-Funktion

5. http://www.diru-beze.de/modulationen/skripte/SuS_W0506/Digitale_Analoge_Modulationen_WS0506.pdf